A Logical Approach to Cooperative Information Systems

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Abstract

"Cooperative information system management" refers to the capacity of several computing systems to communicate and cooperate in order to acquire, store, manage, query data and knowledge. Current solutions to the problem of cooperative information management are still far from being satisfactory. In particular, they lack the ability to fully model cooperation among heterogeneous systems according to a declarative style. The use of a logical approach to model all aspects of cooperation seems very promising.

In this paper, we define a logical language able to support cooperative queries, updates and update propagation. We model the sources of information as deductive databases, sharing the same logical language to express queries and updates, but containing independent, even if possibly related, data. We use the Obj-U-Datalog [5] language to model queries and transactions in each source of data. Such language is then extended to deal with active rules in the style of Active-U-Datalog [4,22], interpreted according to the PARK semantics proposed in [23]. By using active rules, a system can efficiently perform update propagation among different databases. The result is a logical environment, integrating active and deductive rules, to perform update propagation in a cooperative framework.

 $\textit{Key words:}\ \ \text{Deductive databases},\ \text{Active rules},\ \text{Heterogeneous databases},\ \text{Cooperation.}$

1 Introduction

The evolution of current information processing systems towards larger and more heterogeneous systems has made evident the strong need for systems and tools enabling cooperative information system management. Cooperative information system management refers to the capability of several computing systems to communicate and cooperate in order to acquire, store, manage, query data and knowledge. Many application environments, such as workflow management systems, telecommunications network management, digital libraries, health-care provisioning and monitoring, strongly need tools enabling cooperative flows of information among their data management systems.

The development of a cooperative information management system entails addressing different problems, ranging from the differences in hardware/software platforms, to heterogeneity in the database management systems (DBMS), to semantic data heterogeneity, to operational issues such as update propagation and consistency maintenance for related information. Solutions to these problems are provided by efforts in different areas. In particular, hardware/software heterogeneity issues are investigated in the areas of communication networks and operating systems. DBMS heterogeneity (intended as heterogeneity in data models and in query and manipulation languages of the various DBMS) and semantic data heterogeneity are addressed by the area of multidatabase systems [8,40]. Operational issues have been addressed by research on transaction models and mechanisms [19]; also active rules have been applied to the specific issue of update propagation [11,18,26]. A logical approach to enforcing integrity constraints in an heterogeneous environment has been proposed in [12], where temporal logic is extended to explicitly model events and distributed rules. On the contrary, in [44] consistency is not enforced, and different databases are allowed to contain inconsistent information. A supervisory database specifies rules to deal with conflicts as a particular case of integration of information coming from multiple knowledge bases.

Current solutions to the problem of cooperative information management are, however, still far from being satisfactory. In particular, they lack the ability to fully model cooperation among heterogeneous systems according to a declarative style. In most cases, solutions are based on ad-hoc application programs acting as bridges among the various systems. In such a situation, understanding which information and actions a system requires from other systems is very difficult, especially when the number of involved systems is large. If a system is added or removed, new bridges must be defined and analyzed. Moreover, reasoning about the cooperative system as a whole is very difficult. This situation calls for a declarative approach allowing one to fully model all aspects of cooperation and to provide the basis on which properties of cooperative information systems can be proved. To this purpose, the use of a logical approach seems very promising.

In order to model cooperative information management using a logical approach, we should be able to model:

- different sources of data, intended as different databases storing (possibly related) data and intensional knowledge on data;
- cooperative query execution, intended as the ability of several data management systems to collectively provide information to answer user queries;
- updates on data and update propagation among the various data sources.

In this paper, we define a logical language able to support cooperative queries, updates and update propagation. We model the sources of information as deductive databases, sharing the same logical language to express queries and updates, but containing independent, even if possibly related, data.

Several approaches have been proposed integrating logic and updates (see for example [1,2,6,7,13,14,32,34,35,38,45]). The language we propose is based on the U-Datalog language [6], and extends it with support for active rules and with the ability to model heterogeneous databases. U-Datalog has been introduced with the aim of providing a set-oriented logical update language, guaranteeing update parallelism in the context of a Datalog-like language. In U-Datalog, update atoms appear in rule bodies, thus integrating updates and queries and exploiting the power of Datalog queries to select the data to be modified. The execution of a goal (also called a transaction) in U-Datalog is based on a deferred semantics, by which several updates are generated from predicate evaluation, but not immediately executed; rather, they are collected and are executed only at the end of the query-answering process. In U-Datalog, updates are expressed by using constraints. For example, +p(a) states that in the new state p(a) must be true whereas -p(a) states that in the new state p(a) must be false. Each atom solution generates a set of updates.

The language obtained by extending U-Datalog with active rules and support for heterogeneous databases is called *Heterogeneous U-Datalog* (HU-Datalog for short). We define a *HU-Datalog system* as a collection of heterogeneous databases and a set of active rules operating on such databases. In particular, a HU-Datalog system provides the following features:

(1) Multiple sources of data and cooperative query execution. The basic model for each deductive database is a U-Datalog database. As such, it consists of an extensional database (i.e., a set of facts) and an intensional database represented by a set of U-Datalog rules.

The use of U-Datalog provides the ability to model, not only queries, but also updates inside each database. However, U-Datalog alone does not support cooperation among several databases. Each database has no knowledge about the others. In order to enable cooperative query execution, we model each U-Datalog database as an Obj-U-Datalog object [5]. Obj-U-Datalog has been defined as an object-based extension of U-Datalog. Each Obj-U-Datalog object has a state (a set of facts, i.e. an extensional database) and a set of methods (an intensional database).

Objects cooperate through message passing. To this purpose, U-Datalog rules have been extended to request the evaluation of an atom in a different object, i.e. in our case, in a different database. This means that during the evaluation process a database may require the evaluation of a subquery to another database. Message passing is expressed through labeled atoms in rule bodies.

By modeling each database as an Obj-U-Datalog object, information deriving from different sources can be retrieved and updated in a homogeneous way, thus providing cooperative query and update execution. In particular, the use of Obj-U-Datalog makes each source of data aware of the other sources of data belonging to the considered environment.

It is important to note that, if we ignore update atoms, Obj-U-Datalog can be interpreted as an amalgamated knowledge base, as described in [44], based on the classical truth values. The main novelty of our approach is the extension of such amalgamated knowledge bases to deal with updates and, as we will see in the following, actions.

(2) Active rules. In order to exhibit a reactive behavior, active rules in the style of Active-U-Datalog [4,22] are included in the language, according to the PARK semantics [23].

The PARK semantics has been designed with the intent of overcoming the limitations of most previously defined semantics for active rules. In particular, given a set of ECA (Event-Condition-Action) rules, i.e. rules of the form "ON event IF condition THEN action", the PARK semantics satisfies several properties. First of all, it is non-ambiguous, i.e., it always guarantees execution confluence. Moreover, it is flexible with respect to conflict resolution. A conflict is a situation where two or more active rules can be fired and one of these rules requires the insertion of an atom a in the database, whereas at least one of the others requires the deletion of a from the database. A conflict resolution policy is a method to determine which actions should be executed in presence of a conflict and which others should be suppressed. Under the PARK semantics, the conflict resolution policy can be chosen according to specific application requirements. A fixpoint semantics is used to determine the result of the application of a set of active rules. The fixpoint semantics has been chosen because it has a clear mathematical foundation and can be directly implemented. The proposed semantics guarantees the termination of the evaluation process. As it has been pointed out in [23], all other proposed semantics fail to satisfy at least one of these important requirements.

Active-U-Datalog rules are *local* in that each rule refers to a single data source. A rule is triggered when specific events (updates on the considered database) occur. The action is the execution of a set of updates on the considered data source; thus, local active rules can modify only the database into which they are defined.

(3) Update propagation. To support update propagation among heterogeneous databases, global active rules are used. They support consistency

maintenance among different data sources. These rules can fire upon the occurrence of complex events related to all the databases in a HU-Datalog system, can verify conditions spanning all the databases, and can perform actions consisting of updates against all the databases.

The use of active rules for update propagation in the context of heterogeneous databases has been addressed by a few other proposals [11,18,26]. The main difference of our proposal is that the use of the PARK semantics to integrate active and deductive rules makes the approach much more flexible with respect to the problem of conflict resolution. Often, in active database systems, conflicts are solved by assigning priority to rules [27,42,47]. On the contrary, the PARK semantics allows the application programmer to choose the best conflict resolution policy to apply in a particular case. In solving a conflict, information about the structure and the current state of the system can be considered. This is particularly important in a cooperative framework, where update propagation often depends on information distributed among various databases. For example, the resolution policy could include queries on the state of a different database before deciding how to solve a conflict, a behavior that cannot be simulated by using priorities.

Our approach also integrates active and deductive rules. With respect to active rules, the field has produced various results, both commercial [3.17,29,39,41] and academic [15,16,21,25,27,36,42,43,46,47] (the latter usually being more flexible than the former). The semantics we propose for active rules differs from the semantics of most other proposals in that it guarantees termination, polynomial complexity, and confluence; almost all the other proposals fail in satisfying at least one of these requirements. In particular, in order to assign a clear semantics to active rules, several works proposed to integrate active rules in a deductive framework. In giving a semantics to these active-deductive languages, two main approaches emerged. The first one is based on unifying the different paradigms under a common semantics (often by using compilation techniques) [10,33,37,49]. The second one is based on *integrating* specific, different semantics [4,20,22]. In this work, we follow the latter approach. The use of the PARK semantics allows one even in this case to handle updates generated by deductive rules and updates generated by active rules in a uniform way. For example, with respect to U-Datalog and Obj-U-Datalog [5,6], there is no need to define a priori the behavior to be taken when conflicts arise. Thus, our approach can also be seen as an extension of U-Datalog to deal with multiple policies for conflict resolution. As far as we know, no other approach of this kind has been proposed yet in the context of deductive databases.

The paper is organized as follows. In Section 2, we briefly recall the main features of U-Datalog and Object-U-Datalog, on which our proposal is based. Sections 3 and 4 describe respectively the syntax and semantics of HU-Datalog, illustrating them with several examples. In particular, Section 4 introduces an

extension of the PARK semantics to deal with multi-theories and communicating objects. Finally, Section 5 outlines some conclusions, discusses relations between our work and recent work on intelligent agents, and presents future research directions.

2 Overview of U-Datalog and Object-U-Datalog

In this section, we informally present the basic notions of U-Datalog and Obj-U-Datalog. We refer the reader to [5,6] for an extensive description of these languages. Moreover, we assume that the reader is familiar with the basic logic programming concepts [31].

A U-Datalog program (or database) consists of an extensional database EDB, that is, a set of ground atoms, and an intensional database IDB, that is, a set of rules of the following form:

$$H \leftarrow U_1, \dots, U_i, B_{i+1}, \dots, B_n.$$
 $n \ge 1$

where U_j , j=1,...,i, are update atoms and H, B_k , k=i+1,...,n, are atoms, in the usual logic programming sense. An update atom is an atom prefixed by the symbol +, to denote an insertion, or by the symbol -, to denote a deletion. The intuitive meaning of such a rule is "if $B_{i+1},...,B_n$ are true, then H is true and the updates $U_1,...,U_i$ are requested". Predicates defined in the extensional database and predicates defined in the intensional database are disjoint.

A query is a rule with no head. Since, as we will see, U-Datalog queries may generate updates, a U-Datalog query is also called *simple transaction*. A *complex transaction* is a sequence of simple transactions, denoted by $T_1; \ldots; T_n$.

For simplicity, it is assumed that U-Datalog databases are safe. A rule is *safe* if each variable in the head occurs in a non-update atom in the body. Often this condition is relaxed, assuming that the rule is *safe with respect to a query*; in this case, it should be safe when the head is unified with constants in the query.

Example 1 Consider the following U-Datalog program, related to a student database:

IDB: pass(S,E) \leftarrow student(S), exam(E), +passed(S,E).
leave(S) \leftarrow -student(S).

This database contains information about exams (exam predicate), students (student predicate) and the exams they passed (passed predicate). The database can be modified by the predicates pass and leave that, respectively, add the fact that a student passed an exam and delete the student when he/she leaves the school. The first rule is safe, but the second rule is not. However, the second rule is safe with respect to the transaction leave(john). Of course, the second rule can be replaced with leave(S)

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which is always safe.

The semantics of U-Datalog is essentially given in three steps. The first one, called marking phase, generates a set of solutions for a given transaction. Each solution contains a set of bindings and a set of updates. These updates are executed only in the update phase, if they are consistent. A set of updates is consistent if it does not require the insertion and the deletion of the same fact. If the set of updates is not consistent the transaction is aborted and all the updates are discarded. The third step is related to the execution of complex transactions. In this case, the extensional database is updated after each transaction execution. If a transaction aborts, the entire complex transaction aborts as well.

Example 2

Consider again the U-Datalog program introduced in Example 1. The transaction pass(john,math) causes the insertion of the fact passed(john,math) in the database. In this case, bindings for variables contained in the head of the rule are established by the transaction. Now consider the transaction pass(X,Y). In this case, a set of facts is inserted, in particular all the facts of the form passed(s,e), where student(s) and exam(e) belong to EDB. A set of bindings of the form X = s, Y = e is also returned to the user. Note that updates are executed since they are consistent.

Now suppose that students are associated with at least one tutor. Moreover, suppose that two tutors, for some particular reason, need to be assigned each the students of the other. The rules to do that are the following:

```
change(T_1, T_2) \leftarrow -\text{tutor}(S, T_1), +\text{tutor}(S, T_2), \text{tutor}(S, T_1).
change(T_1, T_2) \leftarrow -\text{tutor}(S, T_2), +\text{tutor}(S, T_1), \text{tutor}(S, T_2).
```

Now consider the transaction change (mark, victor). If in the database there exists at least one student -say john- having both mark and victor as tutors, two pairs of inconsistent updates are generated from the previous rules (respectively +tutor(john, mark), -tutor(john, mark) and +tutor(john, victor), -tutor(john, victor)). Thus, an abort is returned and the extensional database is left unchanged.

Obj-U-Datalog [5] has been defined starting from the consideration that a U-Datalog database $EDB \cup IDB$ can be seen as an object where EDB is the

object state and *IDB* represents the set of methods to manipulate (i.e., query and update) such state. Thus an Obj-U-Datalog program consists of a set of object databases, each of which is a U-Datalog database, extended so that distinct databases can refer to each other. Such references are supported by a labeling mechanism. Thus, a method becomes a rule of the form:

$$H \leftarrow U_1, \ldots, U_i, B_{i+1}, \ldots, B_w, db_1 : B_{w+1}, \ldots, db_p : B_z, X_1 : B_{z+1}, \ldots, X_q : B_r.$$

where U_k , k=1,...,i, are update atoms, B_j , j=i+1,...,w, are atoms whose predicates are defined in the object database in which the rule is defined, B_h , h=w+1,...,r, are atoms defined in other object databases. Such object databases are referred to through the use of labels. In particular, db_m , m=1,...,p, are ground labels whereas X_s , s=1,...,q, are variables to which ground labels should be assigned during the evaluation.

Example 3 Suppose that in the school considered in Example 1 there is a library. Library information is maintained in a database which is different from the one containing information about the student. Moreover, a third database stores information about the teachers. The library would like to use some information about students and teachers in order to automatize loans. To accomplish that, the following Obj-U-Datalog program can be defined:

```
school:: student(john).
                                     student(mary).
                                                            student (frank).
          exam(engl).
                                     exam(math).
                                                            exam(phys).
          passed(john,engl).
                                     passed(john, math).
                                     passed(frank, engl).
          passed(mary,phys).
          pass(S,E) \leftarrow student(S), exam(E), +passed(S,E).
          leave(S) \leftarrow student(S),-student(S).
teach::
         prof(william).
                                     prof(isaac).
                                                           prof(eliza).
          teaches(engl,william).
                                     teaches(math, isaac).
          teaches(phys, isaac).
                                     teaches(cs,eliza).
          book(hamlet).
                                     book(principia).
lib::
          sect(engl,hamlet).
                                     sect(phys,principia).
          other_user(helen).
                                     other_user(andrea).
          loan(hamlet, john).
                                     loan(principia, frank).
          user(X) \leftarrow school:student(X).
          user(X) \leftarrow teach:prof(X),teach:teaches(E,X),school:exam(E).
          user(X) \leftarrow other\_user(X).
          deny_loan(B,U) \leftarrow user(U),loan(B,Y).
          deny_loan(B,U) \leftarrow book(B), request(X,U).
          return(B,U) \leftarrow loan(B,U), -loan(B,U).
```

Three object databases have been defined, labeled school, teach, and lib. Predicates defined in the database school have the same meaning of predi-

cates introduced in Example 1. The database teach contains information about teachers and courses they teach; it does not include any deductive rules. The library database lib contains information about users that are neither students nor teachers (other_user predicate), books (book predicate), the section of the library to which the book is assigned (sect predicate), and about who borrowed a book (loan predicate). It also contains rules to specify library users, and the policy to deny a loan and to return a book. In particular, a loan is denied if it concerns a book already on loan, or if it is from a user who was requested to return back a book he borrowed (and has not yet complied).

Notice how rules in the library database use information about students and exams, stored in the school database, and about teachers and their courses, stored in the teacher database, in order to manage loans. In particular, a user can be either a student of the school (first rule defining predicate user) or a professor (teach:prof(X)) teaching a course of the school (second rule defining predicate user) or a person recorded as user inside the database of the library itself (third rule defining predicate user).

As for U-Datalog programs, the semantics of Obj-U-Datalog programs is given in three steps. During the marking phase, however, objects may request the evaluation of some atoms in other objects (specified by labels), thus requiring some form of *context switching*, to solve the query in another object.

3 Syntax of HU-Datalog

In Example 3, we have informally shown how Object-U-Datalog supports interaction among several databases, as long as each of them knows (at least partially) the structure of the others. This condition is too restrictive when applied to the case of completely heterogeneous databases; hence in this case, a different mechanism for integration should be used. This mechanism can be provided by local and global active rules. However, explicit interaction via message passing must also be guaranteed, in order to preserve the expressive power of Object-U-Datalog for those cases in which some databases are indeed tightly coupled. In the following, we present a language called *Heterogeneous U-Datalog* (HU-Datalog for short) having all the previous characteristics. HU-Datalog is thus an extension of Object-U-Datalog.

We consider a many-sorted signature $\Sigma = \{\Sigma_{db}, \Sigma_v\}$, containing only constant symbols. Σ_{db} is the set of database identifiers, whereas Σ_v is the set of constant value symbols. Sets Σ_{db} and Σ_v are disjoint. We consider moreover a set of predicate symbols Π , partitioned into extensional predicate symbols Π^e , intensional predicate symbols Π^i and update predicate symbols Π^u , defined as $\Pi^u = \{+p, -p \mid p \in \Pi^e\}$.

A family of sets of variable symbols for each sort $V = \{V_{db}, V_v\}$ is considered. Terms are defined as usual for each sort of our language, being a term either a constant or a variable. We denote with $Term_{db}$ the set $\Sigma_{db} \cup V_{db}$, and with $Term_v$ the set $\Sigma_v \cup V_v$. We denote with (Π, Σ, V) -atom an atom whose predicate belongs to Π and whose terms are in $\Sigma \cup V$. The notion of substitution is refined to take into account the many-sorted language. A substitution is a pair of functions $\theta = \{\theta_{db}, \theta_v\}, \ \theta_{db} : V_{db} \to Term_{db}, \ \theta_v : V_v \to Term_v$, which maps each variable to a term of the appropriate sort.

Update atoms are extensional atoms prefixed by +, to denote insertion, and by -, to denote deletion. Cooperation among databases in the system is represented by labeled atoms of the form $db: p(\tilde{t})$, where $db \in Term_{db}$, meaning that $p(\tilde{t})$ must be solved in db.

In the following, using the concepts defined above, we formally introduce the notion of HU-Datalog database, HU-Datalog system, and transaction.

Definition 4 (HU-Datalog database)

A HU-Datalog database $DB = EDB \cup IDB \cup AR$ consists of an extensional database EDB, an intensional database IDB and a set of active rules AR. The EDB is a set of ground extensional atoms, called the state of DB.

The IDB is a set of deductive rules of the form

$$H \leftarrow U_1, \ldots, U_i, B_{i+1}, \ldots, B_w, db_1 : B_{w+1}, \ldots, db_p : B_z, X_1 : B_{z+1}, \ldots, X_q : B_r.$$

where

- (1) H is a (Π^i, Σ, V) -atom;
- (2) U_1, \ldots, U_i are (Π^u, Σ, V) -atoms, constituting the update part of the rule;
- (3) B_{i+1}, \ldots, B_w are $(\Pi^i \cup \Pi^e, \Sigma, V)$ -atoms, constituting the unlabeled part of the condition (that is, they refer to the database where the rule is defined);
- (4) $db_1: B_{w+1}, \ldots, db_p: B_z$ are labeled $(\Pi^i \cup \Pi^e, \Sigma, V)$ -atoms referring to specific databases;
- (5) $X_1: B_{z+1}, \ldots, X_q: B_r$ are labeled $(\Pi^i \cup \Pi^e, \Sigma, V)$ -atoms referring to databases that are not yet specified;
- (6) X_1, \ldots, X_q are variables in V_{db} and must appear as arguments of an extensional atom in B_{i+1}, \ldots, B_z .

The update part (U_1, \ldots, U_i) and the set of conditions $(B_{i+1}, \ldots, B_w, db_1 : B_{w+1}, \ldots, db_p : B_z, X_1 : B_{z+1}, \ldots, X_q : B_r)$ cannot be both empty.

The AR is a set of rules of the form

$$E_1, \ldots, E_i, B_{i+1}, \ldots, B_w, db_1 : B_{w+1}, \ldots, db_p : B_z, X_1 : B_{z+1}, \ldots, X_q : B_r \to U_1, \ldots, U_k.$$

where E_1, \ldots, E_i is the event part $(E_j$ is a (Π^u, Σ, V) -atom, $j = 1, \ldots, i)$, $(B_{i+1}, \ldots, B_w, db_1 : B_{w+1}, \ldots, db_p : B_z, X_1 : B_{z+1}, \ldots, X_q : B_r)$ is the condition part $(B_j$ is a $(\Pi^i \cup \Pi^e, \Sigma, V)$ -literal, $j = i+1, \ldots, r)$, and B_j can also be a negative atom (denoted with $\neg p(\tilde{t})$) where negation is understood as negation as failure. U_1, \ldots, U_k is the action part $(U_j$ is a (Π^u, Σ, V) -atom, $j = 1, \ldots, k)$, which cannot be empty. We require two safety conditions for active rules: each variable occurring in a rule head should also occur in the body of the same rule and each variable occurring in a negated literal in the rule body must also occur in some positive literal in the rule body.

The intuitive meaning of a deductive rule is: "if B_{i+1}, \ldots, B_w are true in the database where the rule is defined, B_{w+1} is true in db_1, \ldots, B_z is true in db_p , B_{z+1} is true in the database to which X_1 is instantiated, ..., B_r is true in the database to which X_q is instantiated, then H is true and, as a side effect, the updates U_1, \ldots, U_i are requested". These updates are local, that is, they change the state of the database in which the rule itself is defined. To ensure encapsulation, labeled updates, i.e. updates to be executed on another database, are not allowed in deductive rules. In this way, the state of a database can only be modified through its public interface, which is the set of its intensional predicates. On the contrary, the knowledge contained in the state can be freely queried by any other database.

Databases cooperate by using labeled atoms to request the evaluation of the atom in the context of the database identified by the label. Such label can be either a constant (providing a static communication channel) or a variable (providing a dynamic communication channel). In the latter case, the label identifying the cooperating database is computed by using data contained in some database. Such an approach gives high flexibility for setting up complex, dynamic communication structures.

While deductive rules give deductive power to our framework, active rules allow the system to autonomously react to the current (possibly inconsistent) state and to take appropriate actions to ensure desired properties with respect to the final state. The intuitive meaning of an active rule is: "If the events E_1, \ldots, E_i occur and B_{i+1}, \ldots, B_w are true in the database where the rule is defined, B_{w+1} is true in db_1, \ldots, B_z is true in db_p , B_{z+1} is true in the database to which X_1 is instantiated, ..., B_r is true in the database to which X_q is instantiated, then execute actions U_1, \ldots, U_k ". These active rules are only triggered by local updates and only modify the state of the database which they belong to, since both events and actions are update atoms referring to the database itself. We call them local active rules to distinguish them from the global ones associated with the entire system (see Definition 5). Databases can use local active rules to enforce certain properties of the data, reacting to consistency-breaking changes by updating other data, regardless of the source of changes. Local active rules also simplify the deductive part, allowing one to

centralize certain policies in a single place instead of scattering them between several rules.

Based on the definition of HU-Datalog database, a HU-Datalog system can be defined as follows.

Definition 5 (HU-Datalog system)

A HU-Datalog system $\Xi = \langle \{db_1 :: DB_1, \ldots, db_s :: DB_s\}, AR \rangle$ consists of a set of HU-Datalog databases DB_1, \ldots, DB_s , respectively identified by db_1, \ldots, db_s $(db_i \in \Sigma_{db})$, and of a set of active rules AR of the form:

$$db_{k_1}: E_1, \ldots, db_{k_i}: E_i, db_{h_1}: B_{i+1}, \ldots, db_{h_p}: B_z, X_1: B_{z+1}, \ldots, X_q: B_r \rightarrow db'_1: U_1, \ldots, db'_k: U_k, X'_1: U_{k+1}, \ldots, X'_t: U_m.$$

where $db_{k_1}: E_1, \ldots, db_{k_i}: E_i$ is the event part $(E_j \text{ is a } (\Pi^u, \Sigma, V)\text{-atom}, j = 1, \ldots, i), db_{h_1}: B_{i+1}, \ldots, db_{h_p}: B_z, X_1: B_{z+1}, \ldots, X_q: B_r \text{ is the condition part } (B_j \text{ is a positive or negative } (\Pi^i \cup \Pi^e, \Sigma, V)\text{-atom}, j = i+1, \ldots, r), \text{ and } db'_1: U_1, \ldots, db'_k: U_k, X'_1: U_{k+1}, \ldots, X'_t: U_m \text{ is the action part } (U_j \text{ is a } (\Pi^u, \Sigma, V)\text{-atom}, j = 1, \ldots, m) \text{ which cannot be empty. These rules must satisfy the safety conditions for active rules presented in Definition 4.}$

We call state of a HU-Datalog system Ξ the tuple $\langle EDB_1, \ldots, EDB_s \rangle$, where EDB_i is the extensional database of db_i .

In a HU-Datalog system $\Xi = \langle \{db_1 :: DB_1, \ldots, db_s :: DB_s\}, AR \rangle$ we distinguish a *deductive* part, consisting of the tuple of deductive databases, i.e. $\langle EDB_1 \cup IDB_1, \ldots, EDB_s \cup IDB_s \rangle$, and an *active* part, including the local active rules $\langle AR_1, \ldots, AR_s \rangle$ and the global ones AR.

The rules in AR differ from the local active rules in that all atoms, included events and actions, are labeled. The reason is that those rules refer to the entire system, as they enable update propagation among different sources of data, and they ensure consistency across databases. Since those rules are aware of the structure of each database, they maintain all semantics relationships among relations stored in different databases. If an update is executed with respect to one of such relation, the global rules ensure that the other databases are updated appropriately. Because of their nature, global active rules play the role of mediators [48] in integrating heterogeneous databases.

In order to ensure encapsulation, transactions to be executed in a HU-Datalog system cannot contain update atoms. However their execution may generate updates indirectly, because of the invocation of rules with update atoms in their bodies. Thus a transaction may contain two different kinds of atoms: labeled ones and unlabeled ones. Unlabeled atoms stand for the request of an atom refutation in all the databases composing the system, while labeled atoms are directed to a particular database. We do not restrict labels in transaction to

be constant as required in Obj-U-Datalog, thus allowing dynamic cooperation to be established also at transaction level.

Definition 6 (Transaction) A simple transaction has the form

$$B_1, \ldots, B_w, db_1: B_{w+1}, \ldots, db_p: B_z, X_1: B_{z+1}, \ldots, X_q: B_r.$$

where B_1, \ldots, B_r are $(\Pi^i \cup \Pi^e, \Sigma, V)$ -atoms, db_1, \ldots, db_p are database identifiers and X_1, \ldots, X_q are variables in V_{db} that must appear as arguments of an extensional atom in B_1, \ldots, B_z .

A complex transaction T is a sequence of simple transactions $T_1; \ldots; T_k$.

It should be clear that a transaction provides different functions: the query function, in that it returns a set of bindings, and the update function with a transactional behavior [24]. As we will see in Section 4.3, the transactional behavior ensures that all the updates are executed or, in case of ungroundness, none of them is executed. We always assume that our rules are safe with respect to a transaction.

Example 7 Consider again to the problem of integrating the student database of a school and the loan database of a library. This time, we consider active and fully heterogeneous databases. In particular, each database knows nothing about the structure of the other, as there is no message passing between them (compare this assumption with the ones that the database lib made about the structure of databases school and teach in Example 3).

```
school:: student(john).
                                  student(mary).
                                                           student(frank).
          exam(engl).
                                  exam(math).
                                                           exam(phys).
          passed(john,engl).
                                 passed(john, math).
          passed(mary,phys).
                                 passed(frank, engl).
          pass(S,E) \leftarrow student(S), exam(E), +passed(S,E).
          leave(S) \leftarrow student(S), -student(S).
          -student(S), passed(S,E) \rightarrow -passed(S,E)
lib::
          user (john).
                                  user(mary).
          user(frank).
                                  user(pat).
          book(hamlet).
                                  book(principia).
          sect(engl, hamlet).
                                  sect(phys,principia).
          loan(hamlet, john).
                                  loan(principia, frank).
          deny_loan(B,U) \leftarrow request(X,U), book(B).
          deny_loan(B,U) \leftarrow loan(B,Y), user(U).
          return(B,U) \leftarrow loan(B,U), -loan(B,U).
          -user(U), loan(B,U) \rightarrow +request(B,U).
          -loan(B,U), request(B,U) \rightarrow -request(B,U).
```

Predicates defined in the databases school and lib have the same meaning of predicates defined in Example 3. However, now active rules are included in each database. In particular, the only active rule in school takes care of removing from the database any information related to the exams passed by a student when the student is removed from the database (whatever is the cause of such removal).

In lib, two active rules take care of requesting all the books someone has on loan when he/she is revoked his/her user status, and of removing any pending request for a book when it is returned to the library.

We want to integrate these two databases so that when a student joins the school, he/she is automatically considered a user of the library, and when he/she leaves the school, he/she is removed from the users of the library. Moreover, we want that when a student passes an exam, the library is able to require back all the books the student has on loan from the section corresponding to the passed exam. This behavior can be obtained by adding the following active rules to the global AR set:

```
school:+student(S) \rightarrow lib:+user(S).

school:-student(S) \rightarrow lib:-user(S).

school:+passed(S,E),lib:loan(B,S),lib:sect(E,B) \rightarrow lib:+request(B,S).
```

When Frank passes physics, this information is recorded in the school database by executing the transaction school:pass(frank,phys). The first deductive rule of school causes the insertion of passed(frank,phys) in the school database. Such insertion in turn fires the last global active rule, whose conditions are met by lib:loan(principia,frank) and lib:sect(phys,principia). Therefore request(principia,frank) is added to the library database, disallowing any further loan to Frank until the book is returned. Should John leave the school, the second global active rule would fire, causing the library to remove him from the user list. As part of the local processing activity, the first active rule in the library database would fire, causing the insertion of request(hamlet,john) in that database.

When a book is returned by a transaction lib:return(book,user), the last deductive rule in lib requires the deletion of the loan record. Such deletion fires the last local active rule in lib that removes any pending request for the book.

As another example, consider the problem of a student who wants to move to a different school. We want that exams, passed in the school the student is leaving and common to both schools, are confirmed in the school where he/she is moving. Such information flow can be easily achieved by the following global ARs:

```
\begin{split} & \texttt{school:+move(S,T)} \ \to \ \texttt{school:-student(S),T:+student(S)} \,. \\ & \texttt{school:+move(S,T),school:passed(S,E),T:exam(E)} \ \to \ \texttt{T:+passed(S,E)} \,. \end{split}
```

This solution works if the two schools have the same database schema. In Example 32 we will consider the case of two schools with different but related information. Notice also that the identifier of the database of the new school is obtained from the move predicate and is dynamically used by means of the variable T. \diamondsuit

4 Semantics

The semantics of a HU-Datalog system is given in three steps. In the first step, we compute the model of the deductive part and collect the set of bindings that satisfies the transaction, and the requested updates. This step corresponds to the marking phase in Obj-U-Datalog [5]. However, if the result of this step is a set of inconsistent updates, we do not abort the transaction as Obj-U-Datalog does. Instead, we solve the conflicting updates in the second step. The notion of inconsistency is here extended to sets of labeled updates. A set of labeled updates is *consistent* if it contains no opposite updates labeled by the same database identifier, i.e. $db:+r(\tilde{t})$ and $db:-r(\tilde{t})$.

In the second step, we compute the semantics of the active part of the system, according to the model and the updates collected in the first step. The result of this step is the set of consistent updates requested either from the deductive and/or the active part, in which any conflict has been solved by a parametric policy. Finally, we describe how the two semantics fit together and how we apply the computed updates to the extensional databases of the HU-Datalog databases, thus obtaining the new state of the system.

The observable property of the transaction consists of a set of bindings (the answer), the new system state, and the termination status (commit/abort) of the transaction itself. Actually, our transactions always commit because of the conflict resolution policy of the active part, that solves any inconsistency. Although our semantics never produces aborts by itself, we prefer to keep the flag which indicates the success/failure of a transaction to be consistent with the semantics of U-Datalog and Obj-U-Datalog, and to allow for a smooth extension of our semantics to situations where transactions can go wrong (e.g., implementation-related failures).

The semantics of the deductive part is defined as the collection of the semantics of independent databases, as proposed in [5]. Each database interacts with another database only through explicit context switches, that is, through requests for the evaluation of a subquery sent to the other database. We define a bottom-up semantics for the marking phase, based on a parallel immediate consequence operator. This semantics is based on a composite structure for interpretation in which all the databases in the HU-Datalog system are interpreted simultaneously. For each database, its interpretation is a subset of the set of constrained atoms, \mathcal{B}^{\leftarrow} , defined as follows:

$$\mathcal{B}^{\leftarrow} = \{ H \leftarrow \overline{U} \mid H \text{ is a } (\Pi^i \cup \Pi^e, \Sigma, V) \text{-atom, } \overline{U} \text{ is a set of labeled} \\ (\Pi^u, \Sigma, V) \text{-atoms } \}^{1}$$

The presence of the constrained atom $H \leftarrow \overline{U}$ in the interpretation means that H is true and that its evaluation requires the execution of \overline{U} . All updates in \overline{U} are labeled and their labels refer to the database on which they have to be executed. The labels are constants, whereas the atoms are not necessarily ground.

At this point, an interpretation for a set of HU-Datalog databases DB_1, \ldots, DB_s , identified respectively by db_1, \ldots, db_s , is a tuple of sets $\langle I(db_1), \ldots, I(db_s) \rangle$ where each $I(db_i)$ is a subset of \mathcal{B}^{\leftarrow} that interprets the associated database db_i . The domain $\mathcal{I} = \wp(\mathcal{B}^{\leftarrow})^s$ of all interpretations 2 endowed with the usual order on tuples, induced by the subset order, i.e.

$$\langle I_1, \ldots, I_s \rangle \sqsubseteq \langle I'_1, \ldots, I'_s \rangle$$
 if and only if $I_i \subseteq I'_i$ for all $i = 1, \ldots, s$

is a lattice. Notice that since \mathcal{B}^{\leftarrow} is finite, \mathcal{I} is finite.

The next definition formalizes the intuitive meaning of a rule from the deductive part of the system presented in Section 3. It differs from the consequence operator of Obj-U-Datalog because our resulting interpretation can include constrained atoms with inconsistent updates.

Definition 8 (Immediate consequence operator)

Given a HU-Datalog system $\Xi = \langle \{db_1 :: DB_1, \ldots, db_s :: DB_s\}, AR \rangle$, the immediate consequence operator $T_{\Xi} : \mathcal{I} \to \mathcal{I}$ is defined as follows:

$$T_{\Xi}(I) = \langle T_{db_1}(I), \dots, T_{db_s}(I) \rangle$$

¹ As shorthand, a constrained atom $H \leftarrow$ is simply denoted by H itself.

With D^s we denote the product $\underbrace{D \times \cdots \times D}_{s \ times}$.

where $I \in \mathcal{I}$ and for each i, i = 1, ..., s, we have

$$T_{db_{i}}(I) = \{A \leftarrow \overline{U} \mid H \leftarrow U_{1}, \dots, U_{m}, B_{1}, \dots, B_{n}, db_{k_{1}} : B_{n+1}, \dots, db_{k_{w}} : B_{n+w}, \\ X_{1} : B_{n+w+1}, \dots, X_{p} : B_{n+w+p} \\ is a renamed apart rule of db_{i}, \\ \forall r = 1, \dots, n \ B'_{r} \leftarrow \overline{U}_{r} \in I(db_{i}), \\ \forall q = 1, \dots, w \ B'_{n+q} \leftarrow \overline{U}_{n+q} \in I(db_{k_{q}}), \\ \theta = mgu((B_{1}, \dots, B_{n+w}), (B'_{1}, \dots, B'_{n+w})), \\ \forall t = 1, \dots, p \ B'_{n+w+t} \leftarrow \overline{U}_{n+w+t} \in I(X_{t}\theta), \\ \theta' = mgu((B_{n+w+1}\theta, \dots, B_{n+w+p}\theta), (B'_{n+w+1}, \dots, B'_{n+w+p})), \\ A = H\theta\theta', \\ \overline{U} = \bigcup_{j=1,m} \{db_{i} : U_{j}\theta\theta'\} \cup \overline{U}_{1}\theta\theta' \cup \dots \cup \overline{U}_{n+w+p}\theta\theta' \}^{3}.$$

To build the set of updates, we collect the labeled updates deriving from the resolution of the atoms B_1, \ldots, B_{n+w+p} in the appropriate databases, and the local updates which are labeled by the identifier db_i of the database itself.

The condition requiring that each variable appearing as a label in the body of a rule must appear as argument of an extensional atom of the unlabeled or constant labeled part of the rule, guarantees that the substitution θ is grounding for all such variable labels. Therefore the defined semantics correctly models dynamic channels. Since dynamic labels are no longer visible in the resulting interpretation $T_{\Xi}(I)$, in that they are instantiated on the extensional database, we do not consider them any more in the semantics of the deductive part of HU-Datalog.

It is easy to prove that the operator defined above is monotonic on the lattice $(\mathcal{I}, \sqsubseteq)$ because T_{db_i} is monotonic, $i=1,\ldots,s$. Since the domain is finite, the monotonicity of a function is a sufficient condition for its continuity. Therefore T_{Ξ} is continuous and this allows us to define the fixpoint semantics for a HU-Datalog system as the least upper bound of the chain of the iterated applications of T_{Ξ} starting from the tuple of the extensional databases.

Definition 9 (Deductive Model)

```
Let \Xi = \langle \{db_1 :: EDB_1 \cup IDB_1 \cup AR_1, \dots, db_s :: EDB_s \cup IDB_s \cup AR_s\}, AR \rangle be a HU-Datalog system. The fixpoint semantics \mathcal{F}(\Xi) of \Xi is defined as \mathcal{F}(\Xi) = T_{\Xi}^{\omega}(\langle EDB_1, \dots, EDB_s \rangle), where as usual, T_{\Xi}^{\omega}(\langle EDB_1, \dots, EDB_s \rangle) represents \coprod_n T_{\Xi}^n(\langle EDB_1, \dots, EDB_s \rangle).
```

Notice that the above fixpoint is reached in a finite number of steps due to the finiteness of the domain [9].

 $[\]overline{^3}$ $\overline{U}\theta$ denotes the set of updates obtained by applying the substitution θ to each update in \overline{U} .

Example 10 Consider the following HU-Datalog system with empty sets of local and global active rules.

The least fixpoint computation proceeds as follows:

$$\begin{split} T^0_\Xi(\langle EDB_{\tt m}, EDB_{\tt n} \rangle) &= \langle \{p(a), p(b), q(a)\}, \{z(a), z(b)\} \rangle \\ T^1_\Xi(\langle EDB_{\tt m}, EDB_{\tt n} \rangle) &= \langle \{p(a), p(b), q(a), \ t(a) \leftarrow m : -q(a)\}, \{z(a), z(b)\} \rangle \\ T^2_\Xi(\langle EDB_{\tt m}, EDB_{\tt n} \rangle) &= \langle \{p(a), p(b), q(a), \ t(a) \leftarrow m : -q(a)\}, \{z(a), z(b), \\ & s(a, a) \leftarrow m : -q(a), n : + k(a), \\ & s(b, a) \leftarrow m : -q(a), n : + k(b)\} \rangle \\ T^3_\Xi(\langle EDB_{\tt m}, EDB_{\tt n} \rangle) &= \langle \{p(a), p(b), q(a), \ t(a) \leftarrow m : -q(a), \\ & r(a) \leftarrow m : -q(a), n : + k(a), m : + q(a), \\ & r(b) \leftarrow m : -q(a), n : + k(b), m : + q(b)\}, \\ & \{z(a), z(b), \ s(a, a) \leftarrow m : -q(a), n : + k(a), \\ & s(b, a) \leftarrow m : -q(a), n : + k(b)\} \rangle \\ T^4_\Xi(\langle EDB_{\tt m}, EDB_{\tt n} \rangle) &= T^3_\Xi(\langle EDB_{\tt m}, EDB_{\tt n} \rangle) \end{split}$$

Hence the least fixpoint is $\mathcal{F}(\Xi) = T^3_{\Xi}(\langle EDB_{\mathfrak{m}}, EDB_{\mathfrak{n}} \rangle)$. It is worth remarking that the set of updates in the constrained atom $r(a) \leftarrow m: -q(a), n: +k(a), m: +q(a)$ is inconsistent.

Before introducing the set-oriented semantics we give two auxiliary definitions:

Definition 11 Given a set of bindings b and a transaction T, we define

$$b_{\mid T} = \{(X = t) \in b \mid X \text{ occurs in } T\}$$

Definition 12 Given a substitution $\theta = \{V_1 \leftarrow t_1, \dots, V_n \leftarrow t_n\}$ we define

$$eqn(\theta) = \{V_1 = t_1, \dots, V_n = t_n\}.$$

Now we define the semantics of a simple transaction T with respect to a HU-Datalog system Ξ . As usual in database systems, we give a default set-oriented semantics, that is, the query-answering process computes a set of answers. We denote with $\mathsf{Set}(T,\Xi)$ the set of pairs $\langle bindings, labeled\ updates \rangle$ computed as answers to the transaction T.

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Definition 13 (Simple transaction answer)

Given a HU-Datalog system $\Xi = \langle \{db_1 :: DB_1, \ldots, db_s :: DB_s\}, AR \rangle$ and a simple transaction $T = B_1, \ldots, B_w, db_{k_1} : B_{w+1}, \ldots, db_{k_p} : B_{w+p}, X_1 : B_{w+p+1}, \ldots, X_q : B_{w+p+q}$, then

$$\begin{split} \mathsf{Set}(T,\Xi) &= \{ \langle b, \overline{U} \rangle | \ \forall i = 1, \dots, w \ \exists \ 1 \leq j \leq s. \ A_i \leftarrow \overline{U}_i \in \mathcal{F}(\Xi)(db_j), \\ \forall i = 1, \dots, p \ A_{w+i} \leftarrow \overline{U}_{w+i} \in \mathcal{F}(\Xi)(db_{k_i}), \\ \theta &= mgu((B_1, \dots, B_{w+p}), (A_1, \dots, A_{w+p})), \\ \forall i = 1, \dots, q \ A_{w+p+i} \leftarrow \overline{U}_{w+p+i} \in \mathcal{F}(\Xi)(X_i\theta), \\ \theta' &= mgu((B_{w+p+1}\theta, \dots, B_{w+p+q}\theta), (A_{w+p+1}, \dots, A_{w+p+q})), \\ b &= eqn(\theta\theta')_{|T}, \\ \overline{U} &= \bigcup_{i=1, w+p+q} \overline{U}_i \theta\theta' \ \}. \end{split}$$

Note that, if an atom is not labeled, we look for a database in which this atom can be solved, that is, for a database which includes an instance of the atom in its model. We collect all possible solutions in Set. On the other hand, if the atom is labeled, we restrict our search to the database specified by the label. Such a database is always known since the substitution θ is grounding for variable labels, due to the condition that these variables occur at least in one extensional atom of the unlabeled or constant labeled part of the transaction.

Notice that in Obj-U-Datalog, only pairs with consistent updates are inserted in Set. Instead, we release such restriction, deferring the conflict resolution at the next step (Section 4.2). Moreover, atoms labeled by variables are permitted in transactions, whereas Obj-U-Datalog forbids them.

Example 14 Let r(X), n:z(X) be a simple transaction on the system Ξ described in Example 10. To compute Set, the deductive model of Ξ is exploited, obtaining:

$$\begin{split} \mathsf{Set}((r(X), n : z(X)), \Xi) = \{ & \ \langle \{X = a\}, \{m : -q(a), \ n : +k(a), \ m : +q(a)\} \rangle, \\ & \ \langle \{X = b\}, \{m : -q(a), \ n : +k(b), \ m : +q(b)\} \rangle & \ \} \end{split}$$

Notice that in the first tuple the set of updates is inconsistent. ♦

4.2 Active part semantics

The active part semantics is given following the line of the PARK semantics proposed in [23]. We extend it to deal with labeled atoms and multi-theories. This semantics is well suited to a deferred-update approach, like the one we used in the previous step, and adds much flexibility in that it uses a parametric policy to solve conflicts.

This semantics builds an auxiliary model containing, in particular, the update atoms needed to trigger active rules and to obtain the new system state. To this end, we define a bottom-up operator whose domain is

$$\mathcal{B}^{\pm} = \{db : p(\tilde{t}) \mid db \in \Sigma_{db}, p(\tilde{t}) \in \mathcal{B}\} \cup \{db : +p(\tilde{t}), db : -p(\tilde{t}) \mid db \in \Sigma_{db}, p(\tilde{t}) \in \mathcal{B}, p \in \Pi^e\}$$

where \mathcal{B} is the standard Herbrand Base ⁴. A subset of \mathcal{B}^{\pm} is an *i-interpretation* (where the "i" stands for intermediate). An i-interpretation is *consistent* if it does not contain any pair of opposite updates labeled by the same database identifier, i.e. db:+a and db:-a. This is exactly the consistency definition for sets of labeled updates.

In the following, we denote with d, d' a constant or variable database identifier $(d, d' \in Term_{db})$, with db a constant database identifier $(db \in \Sigma_{db})$, with B, B' a $(\Pi^i \cup \Pi^e, \Sigma, V)$ -atom, with U a (Π^u, Σ, V) -atom, with G a (Π, Σ, V) -atom, and with L a positive or negative (Π, Σ, V) -atom. We sometimes add subscripts to these symbols.

To establish when an active rule can trigger, that is when its event occurs and its condition holds, we introduce the valid function on labeled atoms and i-interpretations.

Definition 15 (Validity) The validity of a ground labeled literal db: a in an *i-interpretation I is defined as follows:*

$$\mathsf{valid}(db : a, I) = \begin{cases} I \cap \{db : p(\tilde{t}), db : + p(\tilde{t})\} \neq \emptyset & \textit{if } a = p(\tilde{t}); \\ I \cap \{db : p(\tilde{t}), db : + p(\tilde{t})\} = \emptyset & \textit{or } db : - p(\tilde{t}) \in I & \textit{if } a = \neg p(\tilde{t}); \\ db : a \in I & \textit{otherwise}. \end{cases}$$

A labeled positive $(\Pi^e \cup \Pi^i, \Sigma, V)$ -atom is valid in I if it belongs to I or if it is inserted by an update in I. A labeled negative $(\Pi^e \cup \Pi^i, \Sigma, V)$ -atom is valid in I if it is deleted by an update in I, or if its positive atom is not valid. A labeled (Π^u, Σ, V) -atom is valid in I if it belongs to I. Notice that both $db: p(\tilde{t})$ and $db: \neg p(\tilde{t})$ can be valid according to the above definition. The intuition behind Definition 15 is that since a labeled positive or negative atom belongs to the condition part of the active rule, its validity must be checked with respect to the derived atoms and also to the inserted and deleted atoms. On the other hand, to represent the occurrence of an event, we require that just the labeled update modeling such an event must belong to the i-interpretation.

 $[\]overline{^4}$ We recall that the standard Herbrand Base is the set of ground positive atoms consisting of all predicate symbols in $\Pi^i \cup \Pi^e$ and constant symbols in Σ .

To solve the condition part of an active rule, we want to use the knowledge contained in the deductive part. However, in exploiting such knowledge, we work under different assumptions with respect to those presented in Section 4.1:

- conditions should be checked by taking into account the requested updates;
- the resolution of a condition should not affect the state of the system.

While the first condition is assured by Definition 15, to fulfill the second condition we remove the update part from the rules of the intensional databases by using the purification operation defined below.

Definition 16 (Purification) Let IDB be the intensional database of a HUDatalog database; we define its purified version \widehat{IDB} as the set of rules

$$B_{i+1}, \ldots, B_w, db_{k_1}: B_{w+1}, \ldots, db_{k_n}: B_z, X_1: B_{z+1}, \ldots, X_q: B_r \to H.$$

such that there exists in IDB a rule

$$H \leftarrow U_1, \ldots, U_i, B_{i+1}, \ldots, B_w, db_{k_1} : B_{w+1}, \ldots, db_{k_p} : B_z, X_1 : B_{z+1}, \ldots, X_q : B_r.$$

Example 17 The purified form of the intensional databases presented in Example 10 is the following:

$$\widehat{IDB}_{\mathtt{m}} = \mathtt{p(X)}, \ \mathtt{n:s(X,Y)} \to \mathtt{r(x)}. \qquad \widehat{IDB}_{\mathtt{n}} = \mathtt{q(X)}, \ \mathtt{p(X)} \to \mathtt{t(x)}. \qquad \mathtt{z(X)}, \ \mathtt{m:t(Y)} \to \mathtt{s(X,Y)}. \qquad \diamondsuit$$

It is worth noting that a query is provable in $\widehat{IDB} \cup EDB$ if and only if it is provable in $IDB \cup EDB$, with the same computed answers. Purification only avoids the side effects of the query evaluation. Also notice that we reversed the direction of the arrow in order to have a uniform notation with active rules.

Both purified and local active rules are transformed in order to have a set of rules having only labeled atoms. This operation is called *labeling* and simply adds to each unlabeled atom of a rule the label db of the database where it is contained.

Definition 18 (Labeling) Given a set of rules R and a database identifier $db \in \Sigma_{db}$, we define the set of labeled rules $\mathsf{lab}(R, db)$ as the set of rules

$$db: L_1, \ldots, db: L_k, d_1: L_{k+1}, \ldots, d_r: L_{k+r} \to db: G_1, \ldots, db: G_m, d'_1: G_{m+1}, \ldots, d'_r: G_{m+s}.$$

such that there exists in R a rule

$$L_1, \ldots, L_k, d_1: L_{k+1}, \ldots, d_r: L_{k+r} \to G_1, \ldots, G_m, d'_1: G_{m+1}, \ldots, d'_r: G_{m+s}.$$

We denote with \mathcal{L} the set of labeled rules. In the sequel, we generically use the term "rules" to refer to both labeled active and labeled purified rules. Notice that Definitions 19, 20, and 21, given a set of rules, only take into account labeled active rules, while subsequent ones work on both kinds of rules.

Now, suppose that given an i-interpretation, several rules are fireable. It may happen that the actions requested by those rules are in conflict. For example, some rules add a certain labeled atom and others remove it from the i-interpretation. In order to obtain a new consistent i-interpretation we prevent one set of rules from firing: if we choose to insert the labeled atom, only the rules adding it are triggered, otherwise only the rules removing it are triggered. Formally, we first define the notion of *conflict*, which consists of a labeled atom and the sets of rules inserting and removing it.

Definition 19 (Conflicts) A pair (r, θ) , where r is a rule and θ is a ground substitution for r is called a rule grounding.

Let P be a set of rules and let I be an i-interpretation for P. Then conflicts(P,I) is a set of maximal triples of the form (db:a,ins,del) such that a is a ground atom, db is a database identifier and ins and del are sets of rule groundings. For each such triple the following conditions must hold:

```
(1) \exists r = d_1: L_1, \ldots, d_n: L_n \to d_{n+1}: U_1, \ldots, d_{n+k}: U_k, \text{ and } r' = d'_1: L'_1, \ldots, d'_m: L'_m \to d'_{m+1}: U'_1, \ldots, d'_{m+s}: U'_s, r, r' \in P, and \exists \theta, \theta' \text{ ground substitutions such that}
```

- $valid((d_i:L_i)\theta, I), i = 1, ..., n,$
- $valid((d'_i: L'_i)\theta', I), i = 1, ..., m,$
- $\exists i, j. \ 1 \leq i \leq k. \ 1 \leq j \leq s. \ U_i = +B, \ U'_j = -B' \ and \ (db:a) = (d_{n+i}:B)\theta = (d'_{m+j}:B')\theta'.$
- (2) For all possible r, r' and θ , θ' , satisfying condition 1 above, $(r, \theta) \in \text{ins}$ and $(r', \theta') \in \text{del}$.

A triple $(db: a, ins, del) \in conflicts(P, I)$ is called a *conflict*. To solve conflicts, a parametric conflict resolution policy is introduced. In the following we call labeled extensional database a set of labeled ground extensional atoms.

Definition 20 (Conflict resolution policy) Let Dom be a domain, recording information about rules (i.e., the priority of the rule, the database the rule belongs to, and anything else which is useful to implement a certain policy). Given a labeled extensional database EDB, a set of rules P, a mapping $f: \mathcal{L} \to Dom$, an i-interpretation I and a conflict c, we define sel(EDB, P, f, I, c) as a total function with codomain {insert, delete}.

The intended meaning of sel(EDB, P, f, I, (db:a, ins, del)) is to choose whether the labeled atom db:a, object of the conflict, should be inserted in or deleted from I, thus effectively choosing which of the conflicting update requests

should prevail. The function f provides information about the rules, which may be useful to implement a certain policy.

Gottlob et al. [23] present a number of commonly adopted policies, and discuss their advantages and disadvantages. We briefly recall here some of them. The principle of inertia states that both the conflicting updates should be discarded, thus leaving EDB in the same state as before with respect to db:a (in our framework, this can be obtained by returning insert if db:a was already in EDB, delete otherwise). The source priority policy determines which update should prevail according to which database the rules requesting such updates come from (in our framework, this can be obtained by using the mapping f which establishes the relation between rules and databases of our system). The rule priority policy, found in systems such as Ariel [27], Postgres [42] and Starburst [47], assumes that each rule has a (static or dynamic) priority associated with it; sel returns insert or delete as needed to preserve the update requested by the highest-priority rule. Other policies, like voting schemes or user queries, are also reasonable, but the final choice is left to the particular application.

Based on the result of the sel policy, we prevent the rule instances in one of the two sets of a conflict from firing, by blocking them according to the following definition.

Definition 21 (Blocked rule instances) Given a labeled extensional database EDB, a set of rules P, a mapping $f: \mathcal{L} \to Dom$, a conflict resolution policy sel, and an i-interpretation I, let

$$\begin{split} X &= \{\mathsf{del} \mid (db \colon a, \mathsf{ins}, \mathsf{del}) \in \mathsf{conflicts}(P, I), \\ &\quad \mathsf{sel}(EDB, P, f, I, (db \colon a, \mathsf{ins}, \mathsf{del})) = \mathsf{insert}\} \\ Y &= \{\mathsf{ins} \mid (db \colon a, \mathsf{ins}, \mathsf{del}) \in \mathsf{conflicts}(P, I), \\ &\quad \mathsf{sel}(EDB, P, f, I, (db \colon a, \mathsf{ins}, \mathsf{del})) = \mathsf{delete}\} \end{split}$$

$$We\ define\ \mathsf{blocked}(EDB,P,f,I,\mathsf{sel}) = \left(\bigcup_{x\in X} x\right) \cup \left(\bigcup_{y\in Y} y\right).$$

An entire rule instance is blocked, instead of the single update responsible for the conflict. This choice has the nice side effect that the set of updates requested by a rule instance exhibits an atomic behavior: either all the updates in the set are executed, or no update at all. Such atomic behavior prevents the risk of an inconsistent database state because of partially-executed actions.

Example 22 Consider the labeled extensional database EDB and the set of rules P given below:

$$EDB = m:t(a,a). m:t(a,b). n:r(a).$$
 $P = r_1 m:t(a,X) \rightarrow m:p(X).$
 $r_2 n:s(X), n:r(X) \rightarrow m:p(X).$
 $r_3 n:+s(X), m:t(X,Y) \rightarrow o:+q(X).$
 $r_4 n:+s(X), m:t(X,Y) \rightarrow o:-q(Y).$
 $r_5 n:+s(X) \rightarrow o:+q(b).$
 $r_6 o:+q(X), m:p(X) \rightarrow m:-t(X,X).$

Given an i-interpretation $I = \{m: t(a,a), m: t(a,b), n: r(a), n: +s(a)\}$, the conflict set is

conflicts
$$(P, I) = \{c_1, c_2\}$$

where

$$c_1 = (o:q(a), \{(r_3, \{X \leftarrow a, Y \leftarrow a\}), (r_3, \{X \leftarrow a, Y \leftarrow b\})\}, \{(r_4, \{X \leftarrow a, Y \leftarrow a\})\})$$

$$c_2 = (o:q(b), \{(r_5, \{X \leftarrow a\})\}, \{(r_4, \{X \leftarrow a, Y \leftarrow b\})\}).$$

Now, consider a function f associating each rule with the database in which the rule is contained. Suppose that the conflict resolution policy sel is such that

$$sel(EDB, P, f, I, c_1) = insert$$
 $sel(EDB, P, f, I, c_2) = delete$

Then, we have that

blocked
$$(EDB, P, f, I, sel) = \{(r_4, \{X \leftarrow a, Y \leftarrow a\}), (r_5, \{X \leftarrow a\})\}.$$

Using the above concepts, the immediate consequence operator on i-interpretations can be defined as follows:

Definition 23 (Immediate consequence operator) Given a set of rules P, a set of blocked rule instances B, and an i-interpretation I, we define $\Gamma_{P,B}(I)$ as the smallest set S satisfying the following conditions:

- $I \subseteq S$:
- If $r = d_1: L_1, \ldots, d_n: L_n \to d'_1: G_1, \ldots, d'_m: G_m \in P$ and θ is a ground substitution for r such that
 - $(r,\theta) \notin B$

$$\text{ valid}((d_i:L_i)\theta,I), i=1,...,n$$

$$then \ \{(d'_1:G_1)\theta,\ldots,(d'_m:G_m)\theta\} \subseteq S.$$

The operator $\Gamma_{P,B}$ is monotonic on the lattice $(\wp(\mathcal{B}^{\pm}), \subseteq)$, hence continuous because $\wp(\mathcal{B}^{\pm})$ is finite.

The main difference of the above operator with respect to the traditional immediate consequence operator of logic programming is that it may happen that some rules are not fired even if their body is valid.

Example 24 Let I be the i-interpretation, P the set of rules, EDB the labeled extensional database, and $B = \mathsf{blocked}(EDB, P, f, I, \mathsf{sel})$ presented in Example 22, then we obtain

$$\Gamma_{P,B}(I) = \{ m: t(a,a), m: t(a,b), n: r(a), n: +s(a), m: p(a), m: p(b), o: +q(a), o: -q(b) \}$$

The use of blocked has prevented the insertion of any conflict in $\Gamma_{P,B}(I)$. \diamondsuit

In general, the application of the function $\Gamma_{P,B}$ to a consistent i-interpretation does not return a consistent i-interpretation. We must appropriately select rules, that is we must build a set of blocked rules B such that the least fixpoint of $\Gamma_{P,B}$ is consistent. Thus, instead of dealing with i-interpretations, the notion of bi-structures is introduced, as in [23], in order to take into account blocked rules.

Definition 25 (Bi-structures) A bi-structure $\langle B, I \rangle$ consists of a set B of rule groundings and of an i-interpretation I. We define an order relation on bi-structures as follows:

$$\langle B, I \rangle \prec \langle B', I' \rangle \quad \Leftrightarrow \quad \begin{cases} B \subset B' & or \\ B = B' & and \ I \subset I' \end{cases}$$

Moreover, given \mathcal{A} and \mathcal{B} bi-structures, $\mathcal{A} \preceq \mathcal{B} \equiv (\mathcal{A} = \mathcal{B} \vee \mathcal{A} \prec \mathcal{B})$. \square

Bi-structures ordered by \leq form a complete partial order because there are only finitely many pairs $\langle B, I \rangle$. On this domain, we can define an operator having a fixpoint, that is used as a model of the active part.

Definition 26 (Δ operator) Given a set of rules P, a mapping $f: \mathcal{L} \to Dom$, a bi-structure $\langle B, I \rangle$, and a conflict resolution policy sel, we define

$$\Delta_{P,f,\mathsf{sel}}(\langle B,I\rangle) = \begin{cases} \langle B,\Gamma_{P,B}(I)\rangle & \textit{if } \Gamma_{P,B}(I) \textit{ is consistent} \\ \langle B \cup \mathsf{blocked}(I^\perp,P,f,I,\mathsf{sel}),I^\perp\rangle & \textit{otherwise} \end{cases}$$

where $I^{\perp} = \{db : p(\tilde{t}) \in I \mid p \in \Pi^e\}$, i.e. the set of labeled extensional atoms contained in I.

The definition of Δ we give here differs from the original in [23] in two respects:

- the i-interpretation consists of labeled ground atoms, because it represents the derived atoms and the updates concerning a set of databases, since we are in a multi-theory context;
- the set of rules P contains not only rules with updates in the right hand side (properly active rules) but also purified rules that derive intensional

knowledge rather than new updates. Notice that this extension does not affect the consistency of i-interpretations, since the purified rules can only add labeled (Π^i, Σ, V) -atoms to the i-interpretation.

The intuitive idea of the Δ operator is that, if no conflict arises, Δ does not change the blocked rules set B, and only the i-interpretation of the bistructure is changed by adding the immediate consequences of the non-blocked rules. On the other hand, as soon as a conflict arises, the conflict is solved via the resolution policy sel and all the blocked rule instances are collected. Then the computation of Δ starts again from the i-interpretation I^{\perp} and the augmented set of blocked rules. The i-interpretation I^{\perp} represents the set of labeled extensional atoms of the system, and we have to resort to it to be sure that the starting point of the new computation does not contain atoms whose validity depends on actions of rule instances that are now blocked. As remarked in [23] for the PARK semantics, this semantics may be viewed as a smooth cycle integrating inflationary fixpoint computation [30] and conflict resolution policies.

The next proposition states that the Δ operator is growing (statement (1)), therefore, for the finiteness of the domain, a fixpoint is reached by iterating Δ a finite number of steps (statement (2)). Furthermore, the i-interpretation of the fixpoint is consistent (statement (3)). The proof is an easy reformulation of the ones presented in [4] and [23].

Proposition 27 Given a set of rules P, a mapping $f: \mathcal{L} \to Dom$, a conflict resolution policy sel, a bi-structure $\mathcal{A} = \langle B, I \rangle$ with I a set of ground labeled extensional atoms, the following statements hold:

- (1) $\mathcal{A} \leq \Delta_{P,f,\mathsf{sel}}(\mathcal{A});$
- (2) there exists k such that $\Delta_{P,f,\mathsf{sel}}^k(\mathcal{A})$ is a fixpoint of $\Delta_{P,f,\mathsf{sel}}$; (3) if $\Delta_{P,f,\mathsf{sel}}^k(\mathcal{A}) = \langle B', I' \rangle$, then $I' = |\mathsf{fp}_I(\Gamma_{P,B'})|^5$ and I' is consistent.

Integrating deductive and active semantics

To assign a semantics to a HU-Datalog system, we must compose the results of the deductive part and active part semantics. We first compute the set of answers in the marking phase (Definition 13). Then we build a set of labeled active and purified rules (Definition 28), and define a mapping $f: \mathcal{L} \to Dom$ and a conflict resolution policy sel in order to apply the Δ operator (Definition 26). From the fixpoint of Δ , we obtain a consistent i-interpretation (Proposition 27), from which the new state of the system is computed (Definition 29).

 $[\]overline{}^{5}$ Ifp_I(f) denotes the least fixpoint of f which is greater or equal to I.

Before presenting the bottom-up semantics of a HU-Datalog system, we need to define the set of rules used to compute the active part semantics, how to incorporate updates in the state and finally, which are the observable properties of a transaction.

The set of rules and the labeled extensional database, used to compute the active part semantics, are defined as follows.

Definition 28 (ρ -set, ε -set)

Let $\Xi = \langle \{db_1 :: EDB_1 \cup IDB_1 \cup AR_1, \dots, db_s :: EDB_s \cup IDB_s \cup AR_s\}, AR \rangle$ be a HU-Datalog system and \overline{U} be a set of labeled updates. We define

$$\begin{split} \rho(\Xi,\overline{U}) &= \left(\bigcup_{i=1,s} \{r \mid r \in \mathsf{lab}(\widehat{IDB}_i \cup AR_i,db_i)\}\right) \cup AR \cup \\ & \left(\{\to db\colon +a \mid db\colon +a \in \overline{U}\} \cup \{\to db\colon -a \mid db\colon -a \in \overline{U}\}\right) \\ \varepsilon(\Xi) &= \bigcup_{i=1,s} \{db_i\colon p(\tilde{t}) \mid p(\tilde{t}) \in EDB_i\}. \end{split}$$

The set $\rho(\Xi, \overline{U})$ contains: the purified rules of the intensional databases and the local active rules, appropriately labeled; the global active rules; and the updates requested from the deductive part represented as rules with neither event nor condition. Therefore this set keeps the intensional knowledge of the system needed to verify the condition part of the active rules, and both local and global active rules needed to maintain local consistency and to propagate updates. On the other hand, $\varepsilon(\Xi)$ contains the extensional knowledge of the system, i.e., the set of all labeled atoms belonging to the state of the system.

The semantics of the previous steps (see Sections 4.1 and 4.2) includes neither the execution of the collected updates nor considers the transactional behavior. We now define a function which given an i-interpretation and the current state of the system, returns the next state obtained by executing the updates in the i-interpretation.

Definition 29 (Updates incorporation) Given a consistent i-interpretation I and a tuple $EDB = \langle EDB_1, \ldots, EDB_s \rangle$, where EDB_i is the extensional database of db_i , we define

$$\mathsf{incorp}(I, EDB) = \langle EDB_1', \dots, EDB_s' \rangle$$

where
$$EDB'_i = (EDB_i \setminus \{a \mid db_i : -a \in I\}) \cup \{a \mid db_i : +a \in I\}.$$

Finally, the observable properties of a transaction we are interested in are the

set of answers, the next system state, and the result of the transaction itself (i.e. Commit or Abort).

Definition 30 (Observables)

An observable Obs is a triple $\langle \mathsf{Ans}, EDB, \mathsf{Res} \rangle$ where Ans is a set of bindings, EDB is a tuple of extensional databases and $\mathsf{Res} \in \{\mathsf{Commit}, \mathsf{Abort}\}$. The set of all observables is denoted by Obs .

Now, we give the semantics of a HU-Datalog system distinguishing two cases, namely, the case of simple transaction and the case of complex transaction.

Definition 31 (HU-Datalog system semantics)

Let $\Xi = \langle \{db_1 :: EDB_1 \cup IDB_1 \cup AR_1, \dots, db_s :: EDB_s \cup IDB_s \cup AR_s\}, AR \rangle$ be a HU-Datalog system, T be a transaction, f be a mapping from the set of rules \mathcal{L} to a certain domain Dom, and sel be a conflict resolution policy. The semantics of a transaction T is denoted by the function $Sem_{\Xi,f,\mathsf{sel}}$ defined as

$$Sem_{\Xi,f,\mathsf{sel}}(T) = \mathcal{S}_{\Xi,f,\mathsf{sel}}(T)(\langle \emptyset, \langle EDB_1, \dots, EDB_s \rangle, \mathsf{Commit} \rangle)$$

where the function $\mathcal{S}_{\Xi,f,\mathsf{sel}}(T):\mathsf{Obs}\to\mathsf{Obs}$ is defined as follows:

If T is a simple transaction, then

$$\mathcal{S}_{\Xi,f,\mathsf{sel}}(T)(\langle \alpha, \xi, \gamma \rangle) = \begin{cases} \langle \emptyset, \xi, \mathsf{Abort} \rangle & \textit{if } \gamma = \mathsf{Abort} \\ \langle \mathsf{Ans}, \mathsf{incorp}(I, \xi), \mathsf{Commit} \rangle & \textit{if } \overline{U} \textit{ ground}, \ \gamma = \mathsf{Commit} \\ \langle \emptyset, \xi, \mathsf{Commit} \rangle & \textit{otherwise} \end{cases}$$

where

$$\begin{split} \mathsf{Ans} &= \{b_j \mid \langle b_j, \overline{u}_j \rangle \in \mathsf{Set}(T,\Xi)\} \\ \overline{U} &= \bigcup_j \{\overline{u}_j \mid \langle b_j, \overline{u}_j \rangle \in \mathsf{Set}(T,\Xi)\} \\ P &= \rho(\Xi, \overline{U}) \\ \langle B, I \rangle &= \Delta_{P,f,\mathsf{sel}}^\omega(\langle \emptyset, \varepsilon(\Xi) \rangle). \end{split}$$

If T is a complex transaction $T_1; \ldots; T_k \ (k \geq 2)$, then

$$\mathcal{S}_{\Xi,f,\mathsf{sel}}(T_1;\ldots;T_k)(Obs) = \mathcal{S}_{\Xi',f,\mathsf{sel}}(T_2;\ldots;T_k)(\mathcal{S}_{\Xi,f,\mathsf{sel}}(T_1)(Obs))$$

where $\Xi' = \langle \{db_1 :: EDB_1 \cup IDB'_1 \cup AR_1 \dots, db_s :: EDB_s \cup IDB'_s \cup AR_s\}, AR \rangle$ and $\langle EDB'_1, \dots, EDB'_s \rangle$ is the new state of the system, that is, the second component of the observable $\mathcal{S}_{\Xi,f,\mathsf{sel}}(T_1)(Obs)$. It is worth remarking that our semantics, unlike the semantics developed in [5], never generates aborts because of conflicts, thus augmenting the successful computations. This is made possible by the active component of our language, and especially by the conflict resolution policy. However, as already discussed, we elect to keep the indication of success (commit/abort) as observable property. Therefore the first case of the above definition, propagating Abort, is formally necessary, although it will be never used in computing the result of a transaction.

Moreover, note that the "otherwise" case discards all changes to the state of the system if a non-ground update is requested. The semantics of a complex transaction is simply given by the sequential composition of the updates generated by its component transactions, since none of them can abort. The state of the system is updated after each simple transaction. Besides, this semantics discards the answer to all but the last transaction, to stay close to the approach in [5].

As already noted, the answer set Set and the Δ fixpoint are computed in a finite number of steps, hence $Sem_{\Xi,f,\mathsf{sel}}$ is computed in a finite number of steps. Actually, it is computable in polynomial time in the size of the state of the system when the mapping f and the conflict resolution policy sel are computable in polynomial time.

Example 32 Consider again the two databases school and lib introduced in Example 7. We want to integrate the school-library system with the database of another school, called sch2. Being in a heterogeneous environment, sch2's structure is quite different from that of school. In particular, the second school admits two types of students, namely undergraduate (undergr predicate) and PhD (phd predicate), and some of its courses are split into two units, with an examination at the end of each unit. A two-units course is considered passed when both the first and the second unit examinations are passed; units must be taken in their natural order.

```
sch2:: undergr(pat). & phd(annie). \\ exam(math,2). & exam(cs,1). \\ units\_passed(pat,math,2). & units\_passed(annie,math,1). \\ units\_passed(annie,cs,1). \\ student(S) \leftarrow undergr(S). \\ student(S) \leftarrow phd(S). \\ passed(S,E) \leftarrow student(S),exam(E,N),units\_passed(S,E,N). \\ pass\_unit(S,E,1) \leftarrow student(S),exam(E,N),+units\_passed(S,E,1). \\ pass\_unit(S,E,2) \leftarrow student(S),exam(E,2),units\_passed(S,E,1), \\ +units\_passed(S,E,2),-units\_passed(S,E,1). \\ \end{aligned}
```

In the database sch2, predicate exam is used to maintain the name of a course

and the number of its units. Predicate units_passed stores, instead, the number of units, of a given course, a student has already passed. Information about courses passed by a student can be obtained from predicate passed, whereas predicate pass_unit updates information about the number of units, of a given course, a student has already passed. For the sake of the example, we do not consider active rules in this database. Notice how sch2 is fully local (no atom is labeled) and totally unaware of the existence of school and lib.

We recall that to integrate the school and lib databases, we introduced the following rules in our global active rules set:

```
\begin{split} & school:+student(S) \rightarrow lib:+user(S). \\ & school:-student(S) \rightarrow lib:-user(S). \\ & school:+passed(S,E),lib:loan(S,B),lib:sect(E,B) \rightarrow lib:+request(B,S). \end{split}
```

These rules express our integration policy: every student of school is a user of lib, and when a student passes an exam, the library requests back any book the student has on loan for the course he/she has just passed. Our integration policy regarding school and sch2 is easily stated: we want that students can move from school to sch2, and in doing so any exam they have passed in the first school is preserved (if applicable) in the new school. The following global active rules state exactly the intended behavior:

```
\begin{split} & school:+move(S,sch2) \rightarrow school:-student(S), sch2:+undergr(S). \\ & school:+move(S,sch2),school:passed(S,E),sch2:exam(E,N) \rightarrow \\ & sch2:+units\_passed(S,E,N). \end{split}
```

These rules are more specialized than the similar ones presented in Example 7 since we cannot simply assume that the other school database has the same structure of the first one. This is a common phenomenon: it is possible to write rules for generic databases if it is possible to assume something about their structure, otherwise specialized rules must be used to perform integration.

We decide also that undergraduate students of sch2 should have access to the library. Such behavior obtained by using the following global active rules:

```
sch2:+undergr(S) \rightarrow lib:+user(S).

sch2:-undergr(S) \rightarrow lib:-user(S).
```

sch2 is more liberal than school regarding loans, so it will not claim back books on loan when a student from sch2 passes an exam.

We still need to introduce the move predicate in school to allow the active rules to fire. This can be obtained by adding

```
transfer(S,T) \leftarrow student(S), +move(S,T).
```

to the IDB of school.

This addition is appropriately placed in school, because a transfer to another school is an event semantically relevant for a student of school, and only the school database can verify whether the student to transfer is actually a student in school. Although the verification is specialized to a given database, as it should be, the actual transfer is not: both transfer and move handle the destination school as a totally generic database. Only in the global active rules the name of the destination is used to perform the appropriate integration.

Now, we can show how the system $\Xi = \langle \{\text{school}, \text{lib}, \text{sch2}\}, AR \rangle$ reacts to the transaction

```
T = school:transfer(john, sch2)
```

The first step computes the fixpoint semantics of the deductive part, according to Definition 9. The result is the following:

```
\mathcal{F}(\Xi) = \langle \{ student(\mathcal{S})^6, exam(\mathcal{E}), passed(john,engl), passed(john,math), \} \rangle
              passed(mary,phys), passed(frank,engl),
              pass(\mathcal{S}, \mathcal{E}) \leftarrow school: + passed(\mathcal{S}, \mathcal{E}), leave(\mathcal{S}) \leftarrow school: -student(\mathcal{S}),
              transfer(\mathcal{S}, \mathcal{D}) \leftarrow school:+move(\mathcal{S}, \mathcal{D}) },
              \{user(S), user(pat), book(K), sect(engl,hamlet),
              sect(phys,principia), loan(hamlet,john), loan(principia,frank),
              deny\_loan(hamlet,S), deny\_loan(hamlet,pat),
              deny\_loan(principia, S), deny\_loan(principia, pat),
              return(hamlet, john) \leftarrow lib:-loan(hamlet, john),
              return(principia,frank) \leftarrow lib:-loan(principia,frank) },
              { undergr(pat), phd(annie), exam(math,2), exam(cs,1),
              units\_passed(pat, math, 2), units\_passed(annie, math, 1),
              units\_passed(annie, cs, 1), student(\mathcal{T}), passed(pat, math),
              passed(annie,cs), pass\_unit(\mathcal{T},\mathcal{G},1) \leftarrow sch2:+units\_passed(\mathcal{T},\mathcal{G},1),
              pass\_unit(annie, math, 2) \leftarrow sch2:+units\_passed(annie, math, 2),
                                                sch2:-units_passed(annie,math.1) }
```

The transaction answer, obtained according to Definition 13, is

$$\mathsf{Set}(T,\Xi) = \{ \langle \emptyset, \{ \mathit{school} : + \mathit{move}(\mathit{john}, \mathit{sch2}) \} \rangle \}.$$

Next, the active part semantics must be computed. The purified intensional databases are

As a shorthand, in the following we write \mathcal{S} to stand for all elements in {john, mary, frank}, \mathcal{E} for all elements in {engl, math, phys}, \mathcal{G} for all elements in {math, cs}, \mathcal{K} for all elements in {hamlet, principia}, \mathcal{T} for all elements in {pat, annie}, and \mathcal{D} for all elements in {school, lib, sch2}.

```
\widehat{IDB}_{\mathtt{school}} = \mathtt{student}(\mathtt{S}), \ \mathtt{exam}(\mathtt{E}) \rightarrow \mathtt{pass}(\mathtt{S},\mathtt{E}).
              student(S) \rightarrow leave(S).
              student(S) \rightarrow transfer(S,T).
\widehat{IDB}_{lib} =
              request(X,U),book(B) \rightarrow deny_loan(B,U).
              loan(B,Y), user(U) \rightarrow deny\_loan(B,U).
              loan(B,U) \rightarrow return(B,U).
\widehat{IDB}_{sch2} =
              undergr(S)\rightarrowstudent(S).
              phd(S) \rightarrow student(S).
              student(S), exam(E,N), units\_passed(S,E,N) \rightarrow passed(S,E).
              student(S), exam(E,N) \rightarrow pass\_unit(S,E,1).
              student(S), exam(E,2), units\_passed(S,E,1) \rightarrow pass\_unit(S,E,2).
Therefore, the \rho-set for \Xi and the updates \overline{U} = \{school: +move(john, sch2)\},\
obtained from Set(T, \Xi), is
 \rho(\Xi, \overline{U}) = \{ school: student(S), school: exam(E) \rightarrow school: pass(S, E), \}
                school: student(S) \rightarrow school: leave(S),
                school: student(S) \rightarrow school: transfer(S, T),
                school: -student(S), school: passed(S, E) \rightarrow school: -passed(S, E),
                lib:request(X, U), lib:book(B) \rightarrow lib:deny\_loan(B, U),
                lib: loan(B, U) \rightarrow lib: return(B, U),
                lib:-user(U), lib:loan(B, U) \rightarrow lib:+request(B, U),
                lib:-loan(B,U), lib:request(B,U) \rightarrow lib:-request(B,U),
                sch2:undergr(S) \rightarrow sch2:student(S),
                sch2:phd(S) \rightarrow sch2:student(S),
                sch2: student(S), sch2: exam(E, N), sch2: units\_passed(S, E, N) \rightarrow
                                                                            sch2: passed(S, E).
                sch2: student(S), sch2: exam(E, N) \rightarrow sch2: pass\_unit(S, E, 1),
                sch2: student(S), sch2: exam(E, 2), sch2: units\_passed(S, E, 1) \rightarrow
                                                                      sch2: pass\_unit(S, E, 2),
                school:+student(S) \rightarrow lib:+user(S),
                school: -student(S) \rightarrow lib: -user(S),
                school:+passed(S,E), lib:loan(S,B), lib:sect(E,B) \rightarrow
                                                                           lib:+reguest(B,S),
                school: +move(S, sch2) \rightarrow school: -student(S), sch2: +undergr(S),
                school: +move(S, sch2), school: passed(S, E), sch2: exam(E, N) \rightarrow
                                                               sch2:+units\_passed(S, E, N),
                sch2:+undergr(S) \rightarrow lib:+user(S),
                sch2:-undergr(S) \rightarrow lib:-user(S),
                \rightarrow school: +move(john, sch2)
The corresponding \varepsilon-set is
 \varepsilon(\Xi) = \{ school: student(john), school: student(mary), school: student(frank), \}
```

```
school: exam(engl), school: exam(math), school: exam(phys),
```

```
school:passed(john, engl), \ school:passed(john, math), \\ school:passed(mary, phys), \ school:passed(frank, engl), \\ lib:user(john), \ lib:user(mary), \ lib:user(frank), \ lib:user(pat), \\ lib:book(hamlet), \ lib:book(principia), \ lib:sect(engl, hamlet), \\ lib:loan(hamlet, john), \ lib:loan(principia, frank), \\ sch2:undergr(pat), \ sch2:phd(annie), \ sch2:exam(math, 2), \\ sch2:exam(cs, 1), \ sch2:units\_passed(pat, math, 2), \\ sch2:units\_passed(annie, math, 1), \ sch2:units\_passed(annie, cs, 1) \\ \}
```

We assume an inertial conflict resolution policy (see Section 4.2) that does not use the mapping $f: \mathcal{L} \to Dom$. Therefore we leave f undefined. The computation of the system semantics through the Δ operator proceeds as follows:

```
\begin{split} \Delta^0_{\rho(\Xi,\overline{U}),f,\mathsf{sel}}(\langle\emptyset,\varepsilon(\Xi)\rangle) &= \langle\emptyset,I_1 = \varepsilon(\Xi)\rangle \\ \Delta^1_{\rho(\Xi,\overline{U}),f,\mathsf{sel}}(\langle\emptyset,\varepsilon(\Xi)\rangle) &= \langle\emptyset,I_2 = I_1 \cup \{\ school:pass(\mathcal{S},\mathcal{E}),\\ school:leave(\mathcal{S}),\ school:transfer(\mathcal{S},\mathcal{D}),\\ lib:deny\_loan(hamlet,\mathcal{S}),\\ lib:deny\_loan(hamlet,pat),\\ lib:deny\_loan(principia,\mathcal{S}),\\ lib:return(principia,frank),sch2:student(\mathcal{T}),\\ school:+move(john,sch2)\ \} \rangle \\ \Delta^2_{\rho(\Xi,\overline{U}),f,\mathsf{sel}}(\langle\emptyset,\varepsilon(\Xi)\rangle) &= \langle\emptyset,I_3 = I_2 \cup \{\ sch2:pass\_unit(\mathcal{T},\mathcal{G},1),\\ sch2:pass\_unit(annie,math,2),\\ school:-student(john),\ sch2:+undergr(john),\\ sch2:+units\_passed(john,math,2)\ \} \rangle \end{split}
```

Now, a conflict arises. Indeed:

```
I' = \Gamma_{\rho(\Xi,\overline{U}),\emptyset}(I_3) = I_3 \cup \{ school: -passed(john, engl), \\ school: -passed(john, math), \ lib: -user(john), \\ lib: +user(john) \}.
```

The labeled atom object of the conflict is lib: user(john). The rule that tries to insert it is $r_1 = \text{sch2}$: +undergr(S) \rightarrow lib: +user(S), with grounding substitution $\{S \leftarrow john\}$; the rule that tries to remove it is $r_2 = \text{school}$:-student(S) \rightarrow lib:-user(S), with the same grounding substitution. Both rules belong to the AR set. Since we are using the inertial conflict resolution policy, $\text{sel}(\varepsilon(\Xi), \rho(\Xi, \overline{U}), f, I', (lib:user(john), \{(r_1, \{S \leftarrow john\})\}, \{(r_2, \{S \leftarrow john\})\}) = \text{insert}, thus blocking <math>(r_2, \{S \leftarrow john\})$. The computation proceeds without further conflicts, restarting from the bi-structure $\langle B = \{(r_2, \{S \leftarrow john\})\}, \varepsilon(\Xi) \rangle$.

$$\begin{split} &\Delta^3_{\rho(\Xi,\overline{U}),f,\mathsf{sel}}(\langle\emptyset,\varepsilon(\Xi)\rangle) = \langle B,I_1\rangle \\ &\Delta^4_{\rho(\Xi,\overline{U}),f,\mathsf{sel}}(\langle\emptyset,\varepsilon(\Xi)\rangle) = \langle B,I_2\rangle \\ &\Delta^5_{\rho(\Xi,\overline{U}),f,\mathsf{sel}}(\langle\emptyset,\varepsilon(\Xi)\rangle) = \langle B,I_3\rangle \\ &\Delta^6_{\rho(\Xi,\overline{U}),f,\mathsf{sel}}(\langle\emptyset,\varepsilon(\Xi)\rangle) = \langle B,I_4=I_3\cup\{\ school:-passed(john,engl),\\ & school:-passed(john,math),\ lib:+user(john)\\ &\rbrace\rangle \\ &\Delta^7_{\rho(\Xi,\overline{U}),f,\mathsf{sel}}(\langle\emptyset,\varepsilon(\Xi)\rangle) = \Delta^6_{\rho(\Xi,\overline{U}),f,\mathsf{sel}}(\langle\emptyset,\varepsilon(\Xi)\rangle) \end{split}$$

Hence the fixpoint is reached.

After performing update incorporation (Definition 29), the new state EDB' of the system is

```
EDB'_{school} = student(mary).
                                    student(frank).
             exam(engl).
                                    exam(math).
                                                             exam(phys).
             passed(mary,math).
                                    passed(frank, engl).
             move(john,sch2).
EDB'_{\mathtt{lib}} =
             user(mary).
                                    user(john).
             user(frank).
                                    user(pat).
             book(hamlet).
                                    book(principia).
             sect(engl, hamlet).
                                    sect(phys,principia).
             loan(john,hamlet).
                                    loan(principia, frank).
EDB'_{\mathtt{sch2}} =
            undergr(pat).
                                    undergr(john).
                                                             phd(annie).
             exam(math, 2).
                                    exam(cs,1).
             units_passed(pat,math,2).
                                              units_passed(annie,math,1).
             units_passed(annie,cs,1).
                                              units_passed(john,math,2).
```

By looking at the new state of the system, we can see that now John is an undergraduate student of school sch2 and he is still a user of the library. However, he lost his English exam because such a course is not taught in the school sch2.

5 Conclusions and future work

and the final observable is $Obs' = \langle \emptyset, EDB', \mathsf{Commit} \rangle$.

In this paper, we defined a logical language supporting cooperative queries, updates, and update propagation. We model the sources of information as deductive databases, sharing the same logical language to express queries and up-

dates, but containing independent, even if possibly related, data. The language used to model data sources has been obtained by extending the Obj-U-Datalog language [5] to deal with active rules in the style of Active-U-Datalog [4,22], whose semantics has been defined according to the PARK semantics proposed in [23]. The use of Obj-U-Datalog enables cooperative query answering and distributed transaction execution among different data sources, whereas active rules perform update propagation and consistency maintenance. The proposed framework results in a uniform integration of active and deductive rules to model cooperation.

This work can be extended in several ways. A first important question is related to the definition and analysis of properties concerning execution of distributed queries, transactions and active rules. A second direction concerns various extensions to the proposed language, such as the introduction of negation in the deductive part of a HU-Datalog system and constructs for modeling more complex events. Finally, since a HU-Datalog database, when ignoring updates and active rules, represents a particular amalgamated knowledge base, as defined in [44], another interesting topic is the extension of the general amalgamated knowledge base framework to deal with updates and actions.

As concluding remark we would like to comment on the relations between our work and recent work on agent technology. Even though it is difficult to precisely define what an agent is, we can consider an agent as "a self-contained program capable of controlling its own decision-making and acting, based on its perception of its environment, in pursuit of one or more objectives" [28]. More specifically, an agent should be characterized by a number of key properties including: social ability - referring to the ability to interact with other agents and/or humans; autonomy - referring to the control an agent should have over its own actions and internal state; reactivity - referring to the ability of an agent to react to changes that occur in the agent state and in the environment. The object model we have developed in this work satisfies all these key properties. In particular, in our model, an object is characterized by a state and a mechanism to perform queries and modifications to the object state. Moreover, objects can communicate with other objects, also according to unanticipated modalities, through the use of labeled atoms. Since labels can also be variables, both static and dynamic communication channels among objects are supported. Finally, objects are able to react to events through the use of active rules. We plan to investigate the use of our object model as a foundation for agent technology dealing with intelligent information management.

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